THE General Radio EXPERIMENTER

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ELECTRICAL MEASUREMENTS AND THEIR INDUSTRIAL APPLICATIONS

DECADE VOLTAGE DIVIDER NOW AVAILABLE

Also

IN THIS ISSUE

● **THE TYPE** 654-A Voltage Divider, always a useful laboratory device, was discontinued during the war in order to concentrate production facilities on more urgently needed items. We have had a number of requests for this item and are glad to announce that it is again available from stock.

As shown in the accompanying photograph, this voltage divider has three dials, giving division factors of 0.1, 0.01, and 0.001, respectively. Voltage ratios between 0.001 and 1.000 can be obtained in steps of 0.001 with an accuracy of ±0.2%. The division is accomplished by the equivalent of two three-dial decade resistance boxes, so connected that when resistance is taken from one box it is added to the other to maintain the total resistance constant at 10,000 ohms. This action is accomplished through the use of two Type 510 Decade-Resistance Units operated from each control knob by means of a chain drive. All resistors are wound with an alloy wire of such characteristics that no difficulty due to thermal emf will be encountered in direct-current measurements.

The Type 654-A Voltage Divider is currently priced at $100.00 plus 10%. Complete specifications will be sent on request.

Panel view of the decade voltage divider.

THOSE IRON-CORED COILS AGAIN

Since publication of the article on iron-cored coils in the *Experimenter* a few years ago[1], considerable use has been made during the intervening years of the theory developed therein. It is not surprising that there have emerged, during that period, a number of ways in which the use and application of the theory may be simplified.

PART I

INTERCHANGEABILITY OF PERMEABILITY AND FREQUENCY

One of the more significant improvements has to do with the method of securing empirically the information needed for a particular lamination structure. This needed information consists of the maximum storage factor Q of a full coil wound on the lamination structure, the frequency at which it occurs, and the law of variation of inductance with center-leg air gap.

Theory

If reference be made to page 9 in the Appendix portion of the March, 1942, article, it will be seen that for a given structure D_c varies inversely while D_e varies directly as the product $f\mu$. Hence, if frequency is held constant, dissipation factors D_c (copper loss) and D_e (eddy-current) are functions solely of effective permeability μ. Thus,

$$D_c = \frac{c'}{\mu} \quad \text{and} \quad D_e = e'\mu \qquad (24)\ \&\ (25)$$

where,

$$c' = \frac{10^9 \rho_c tl}{8\pi^2 fSA\alpha} \quad \text{and} \quad e' = \frac{2\pi^2\delta^2 f}{3\rho_i 10^9} \qquad (26)\ \&\ (27)$$

(Measurements are postulated at very small B, where hysteresis loss is negligible.) Consequently, the same shape of Q curve will be obtained at constant frequency and varying permeability as with constant permeability and varying frequency.

The Q of the coil will vary from a low value with the core interleaved up through a maximum value at some intermediate air gap and down again to a low value at a large air gap.

Technique

It will be necessary to choose an appropriate frequency at which to make the measurements, in order to be sure that the peak occurs somewhere near the geometric center of the inductance range covered. While experience will usually provide a guide to the selection of the measuring frequency, a more reliable one is provided by the expression for f_m and a few measurements on various types of cores.

The frequency at which the Q of a coil is a maximum is given by the expression

$$f_m = \frac{10^9}{4\pi^2\mu\delta}\sqrt{\frac{3\rho_c\rho_i tl}{SA\alpha}} \qquad (23)$$

These frequencies are made lower as the size of the core is increased, as the laminations are made thicker, as the permeability of the core is increased, or as the resistivity of the core is decreased. Both of the last two reasons, for instance, operate to make an A-metal core require a lower measuring frequency than a silicon-steel one.

If the frequency chosen were that corresponding to Q_m for an interleaved core, the measured points would all fall at and on one side (the left) of the maximum, while for the frequency corresponding to no iron (air core) the distribution of points would be on the other side of the maximum. A convenient frequency is the geometric mean of the two values of f_m corresponding to Q_m for the interleaved core and Q_m for the air core. This frequency corresponds to a μ which is the geometric mean between that of the interleaved material and the effective value for the air core.[2]

[1] P. K. McElroy and R. F. Field, "How Good is an Iron-Cored Coil?" *General Radio Experimenter*, March, 1942.

[2] This value is greater than unity for at least two reasons: (1) The turns are not concentrated in a single layer, but fill the whole window, thus yielding a larger inductance; and (2) the flux path external to the center leg is a much greater fraction of the total.

For example, measurements of a number of coils with cores of 4% silicon steel indicate that the ratio of inductance, and hence equivalent permeability, between air core and full-interleaved core is about 65. For an initial permeability of the core material of 470, the equivalent air core permeability is then about 7. Using the geometric mean of 60 in Equation 23 gives the optimum frequency for taking data for the humped Q curve. For A-metal, the permeability values are 2500 and 7, giving a geometric mean of about 130.

The following table indicates the optimum measuring frequencies for gathering data for humped curves as functions of the lamination size and material (in the order of size). The dimensions of the four standard GR lamination shapes here discussed appear in Figure 1.

Lamination	4% Silicon Steel	A-Metal
746	1805 cps	782 cps
345	823	439
485	612	327
365	462	—

These figures can be compared with the notes following shortly below and with the distribution of plotted points on the curves in Figures 2 to 11.

In the examples illustrated by those figures, various combinations of lamination shape and material appear. It should be noted, in connection with all the curves mentioned here, that a point representing measurements on an air-core coil (ferro-magnetic material completely removed) generally lies right on the curve which goes through the points representing conditions where there is some ferromagnetic material in the circuit. These points are identified on the plots by an adjacent letter "A." A letter "B" on each plot will be placed adjacent to the point representing a butt joint in the center leg (interleaved outside legs), and a letter "I" beside the point representing the completely interleaved condition.

Some notes correlating the symmetry of the humped curves with the measuring frequency are given in the captions. Unless otherwise stated, fair symmetry was achieved.

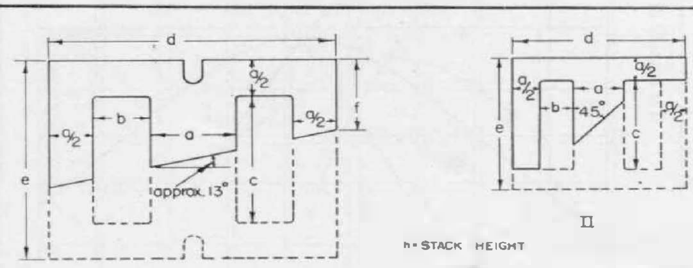
Plotting the Curves

The plots are all made with values of Q as ordinates, and values of inductance as abscissae. Since the only information to be extracted from the humped curves is the maximum Q reached, there is no necessity to reduce the data so that plotting can be done against the effective permeability to which the inductance is, by definition, strictly proportional.

Each of Figures 2 to 8 will be seen to carry two additional curves. The second curve, which slopes downward from left to right, is obtained (from the original data) by plotting measured center-leg air gap in mils against measured inductance (at the initial permeability level)[3]. These curves would have a constant downward 45° slope (inductance inverse with air gap), were it not for the effects of fringing, of the reluctance of the magnetic material, and of the equivalent reluctance of a butt joint.

The third curves, sloping upward to the right, are derived from the second ones. For them, the ordinates are the same as for the second ones, namely, measured air gap. The abscissae, however, are frequencies in cycles according to the scale at the top of the figure. These frequencies are the frequencies at which the maximum Q occurs for various air gaps. This curve can be used to decide what air gap will give the best compromise between stability of inductance against voltage changes and high Q at the desired frequency. When the air gap has been decided upon, the frequency at which the maximum Q will occur can be read from the curve.

[3]No points for air-core, interleaved, or butt joints are included, since it is impossible to assign specific values of air gap to these conditions on the log-log chart.



LAMINATION	DWG.	a	a/2	b	c	d	e	f	h	STD. STACK
746	II	344	172	250	750	1-3/16	1-3/32	—	23/32	2 1
345	I	750	375	531	1-1/8	2.5625	1-7/8	5/8	3/4	SQUARE
485	I	15/16	15/32	21/32	1-19/32	3-3/16	2-17/32	29/32	15/16	SQUARE
365	I	1-3/16	19/32	25/32	2-1/8	3-15/16	3-5/16	1-3/16	1-3/16	SQUARE

Figure 1. Dimensions of core laminations used in measurements.

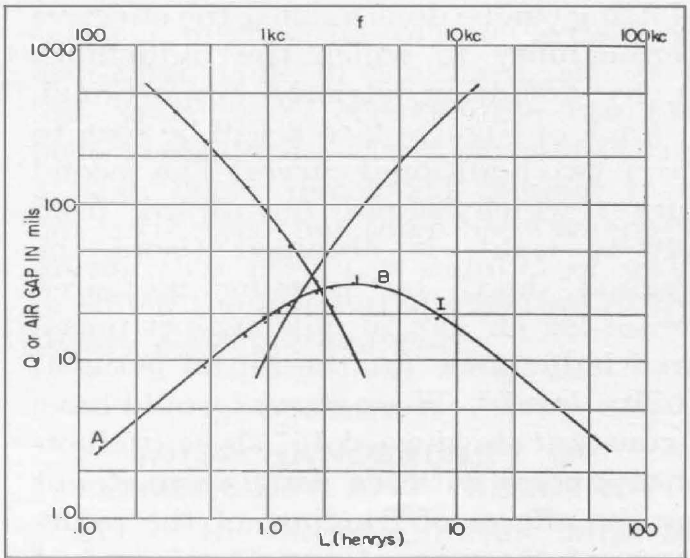


FIGURE 2

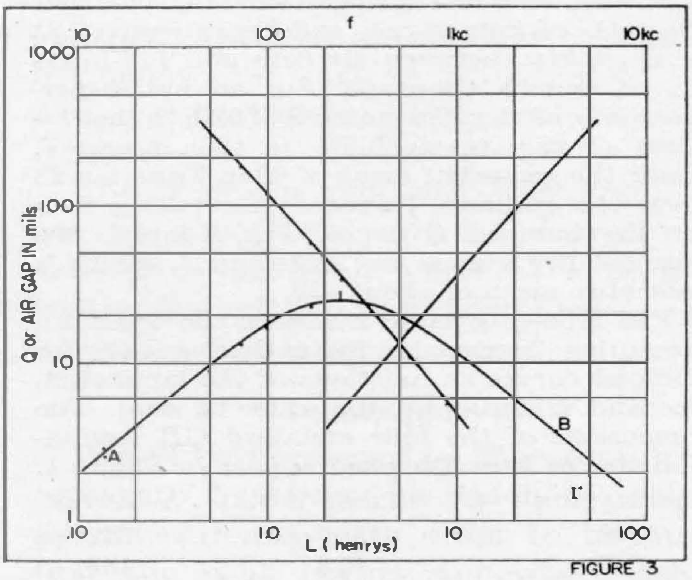


FIGURE 3

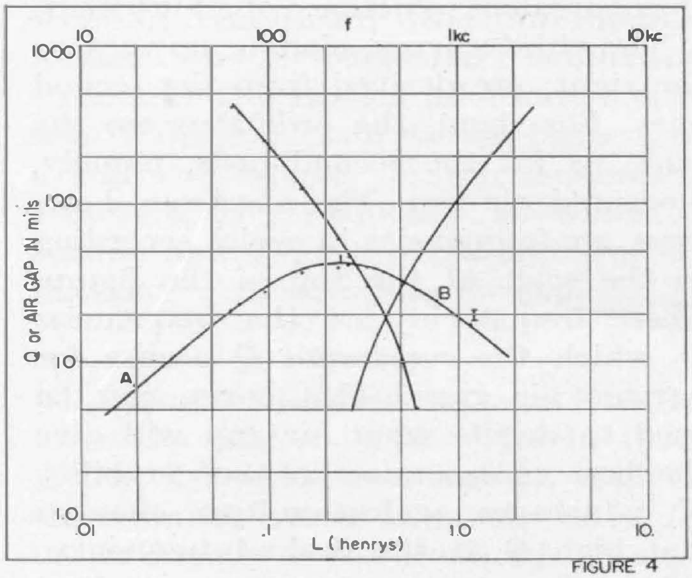


FIGURE 4

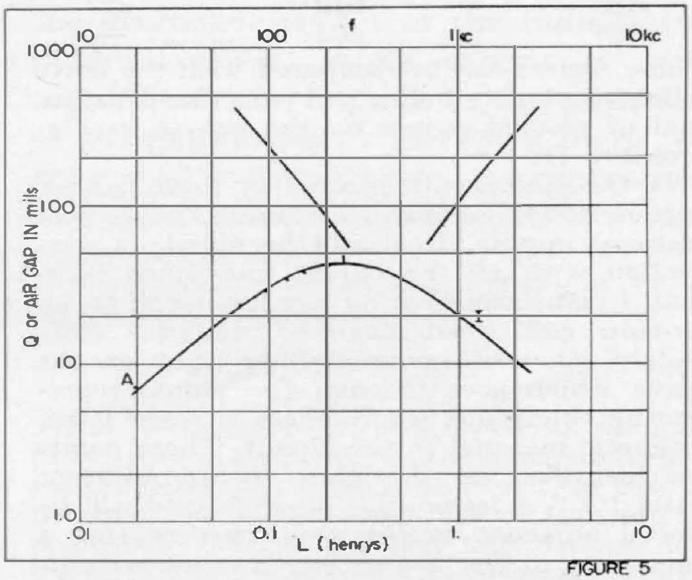


FIGURE 5

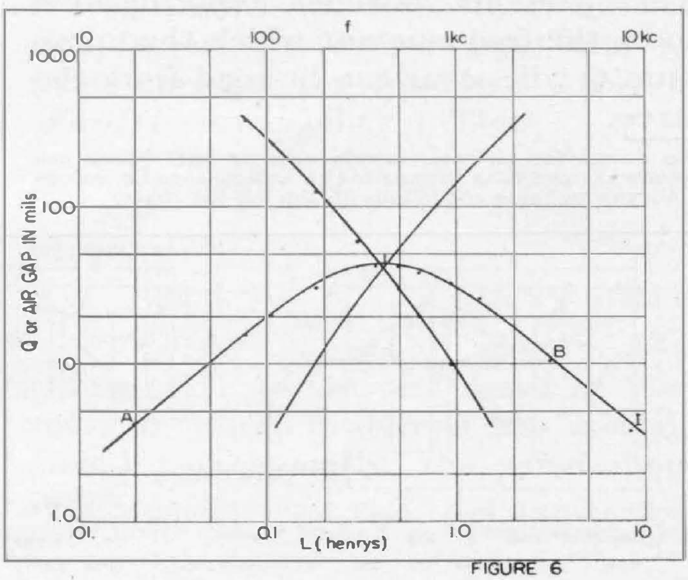


FIGURE 6

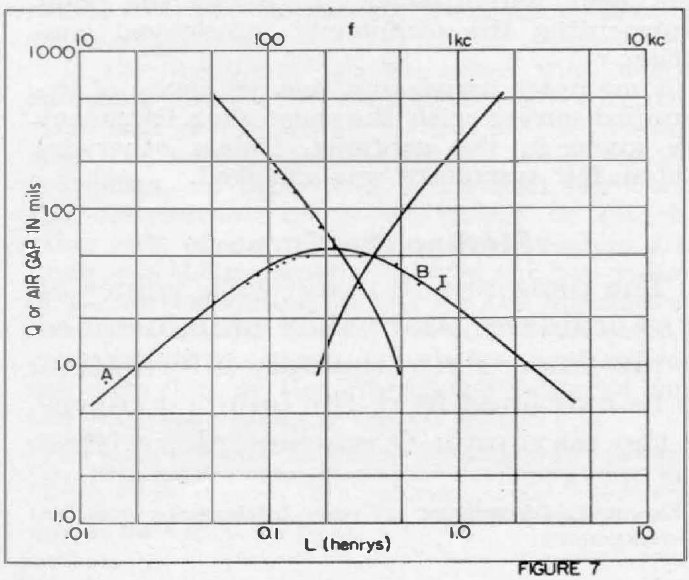


FIGURE 7

If, now, the center point of a humped cardboard template be laid on the chart at the point where a vertical line corresponding to the frequency indicated as above intersects a horizontal line representing the Q_m characteristic of this structure independent of air gap (passing through the peak of the original humped curve), one can read the Q of coils on this structure (having the chosen air gap) at any other frequency.

To derive the third curve from the second, use is made of the relationship that Q_m occurs at a frequency which is inverse with effective permeability, i.e., with inductance. The product, then, of inductance and frequency for Q_m at all points is the same. The value of this product can be obtained by multiplying the frequency of measurement by the inductance at which maximum Q of the humped curve occurs. If Figure 8 is taken as an example, the maximum of the humped curve occurs at 0.245 henries, and the measuring frequency is 400 cycles. The product of the two is 98. The air gap corresponding to 0.245 henries is 68 mils at 400 cycles. To obtain a point on the third curve corresponding to any point on the second curve, divide the abscissa for the second curve into 98, from which will be obtained the abscissa for the third curve. The ordinate is the same for both. The end result is two curves, mirror images of one another.

Figure 2. A coil wound on a small, or postage-stamp, core (GR 746) of 29-gauge 4% silicon-steel laminations with an 11/32" wide center leg stacked 23/32" high was measured at 1 kc, but the plot was quite lopsided, with most of the points on the low-inductance side of the peak. The frequency should have been higher.

Figure 3. A coil on the same core as that of Figure 2, employing 29-gauge A-metal laminations, was measured at 1 kc, which was nearly the correct frequency.

Figure 4. A coil on a 345 core of 26-gauge 4% silicon-steel laminations, having a 3/4" square center leg, was measured at 700 cycles. The laminations had a blunt-angled nose. (See Figure 12.)

Figure 5. The same coil as in Figure 4, but with square-nosed laminations. Unfortunately, they were available only with a center-leg air gap no smaller than 1/16". Therefore, there are no points between approximately 0.25 henries (corresponding to 1/16" air gap) and 1.3 henries (corresponding to completely interleaved laminations). The two sloping curves are short for the same reason. Within the range where both have points, the curves of Figures 4 and 5 should be and actually are sensibly in the same locations:

Figure 6. A coil on the same size core, but made from 29-gauge A-metal laminations, was measured at 400 cycles.

Figure 7. A coil on a 485 core of 26-gauge 4% silicon-steel laminations, having a 15/16" square center leg, was measured at 400 cycles.

Figure 8. A coil on the same size core, but made from 29-gauge A-metal laminations, was measured at 400 cycles.

Figure 9. A coil of 8800 turns of No. 30 enamel on 365 core of 26-gauge 4% silicon-steel laminations, having a 13/16" square center leg, was measured at 400 cycles.

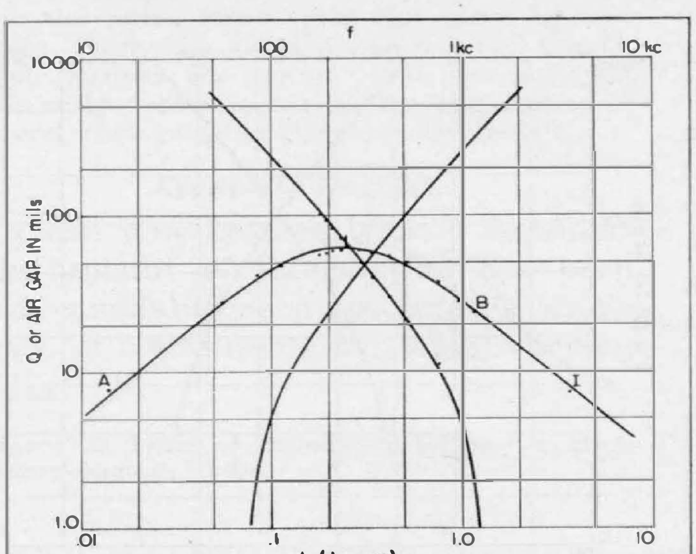


FIGURE 8

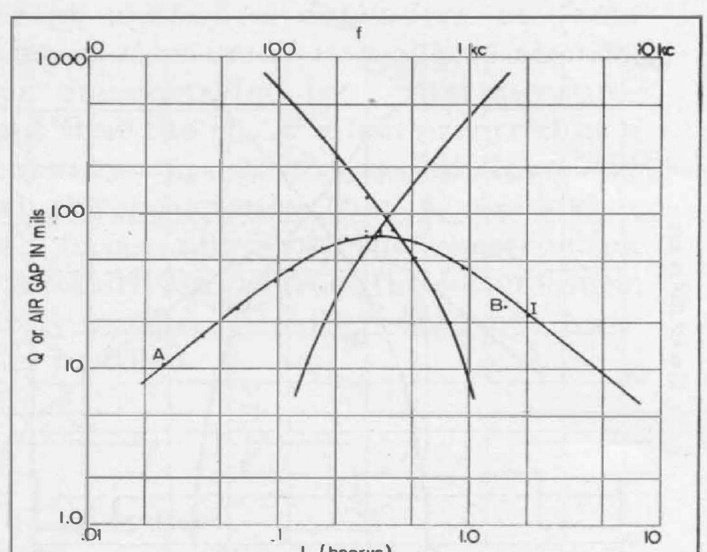


FIGURE 9

Comparison of Graphic Results With Theory

The following equations are repeated from the March, 1942, article:

$$L = \frac{4\pi N^2 \mu A \alpha}{10^9 l} \qquad (6)$$

$$f_m = \frac{10^9}{4\pi^2 \mu \delta} \sqrt{\frac{3\rho_c \rho_i t l}{S A \alpha}} \qquad (23)$$

If they are combined by eliminating μ, the following equation results:

$$L = \frac{N^2}{\pi \delta f_m} \sqrt{3\rho_c \rho_i \frac{t}{S} \cdot \frac{A\alpha}{l}} \qquad (28)$$

This equation yields a figure for the value of inductance at which the top of the humped curve should occur for the particular measuring frequency employed. This is compared in the following table with the value obtained graphically from each plot.

		L		Q	
lam.	mat.	calc.	act.	calc.	act.
746	SS	2.76	2.95	28.0	30
746	A	2.59	2.43	26.3	25
345	SS	0.219	0.24	40.6	42
345	A	0.442	0.42	46.9	43
485	SS	0.198	0.216	54.9	54
485	A	0.228	0.244	63.4	63
365	SS	28.7	33.5	72.1	71.5

In the same way the following equation

$$Q_m = \frac{1}{\delta} \sqrt{\frac{3\rho_i S A \alpha}{\rho_c t l}} \qquad (22)$$

gives the maximum height which should be reached by the humped Q curve. This likewise is compared with the graphically obtained value in the table.

It will be noted that the agreement between theory and measurement is fairly good when the vagaries of ferromagnetic materials are borne in mind.

Air-core Point

The fact that the point representing measurements on an air-core coil falls on the curve as closely as other points may at first thought seem strange. However, this point represents the extreme limit of low permeability where copper loss is the sole factor in determining Q. For this condition the flux cutting the copper is identical with that for an interleaved core.

In order to demonstrate that this point belongs on the curve, extra points to define the curve were taken for Figure 9. The two points to the right of point A represent air gaps of 1¾ inches and 1⅛ inches, respectively. The third point corresponds to a ⅝-inch gap, the longest normally measured.

Eddy Currents in Copper

It will be noted that a number of points in the middle of the Q curves (that is, for moderate-length air gaps) of Figures 10 and 11 do not fall on the curve. This is attributable to increased eddy-current losses in the copper.

Figure 10. A coil wound with 972 turns of No. 20 enamel, measured at 400 cycles.

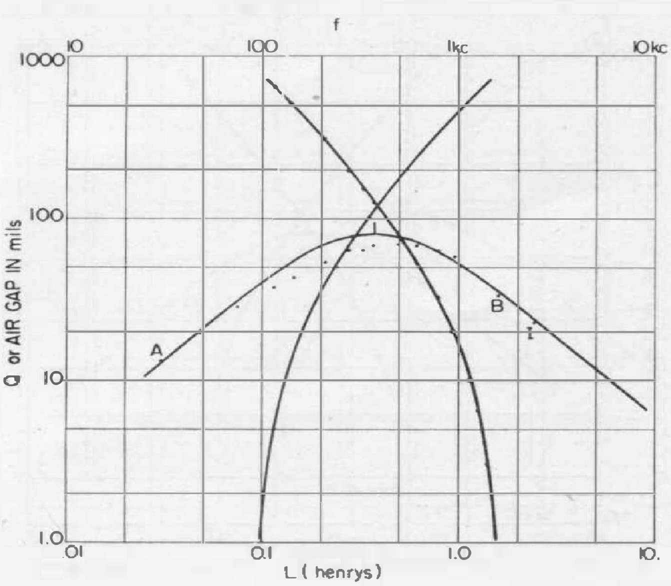
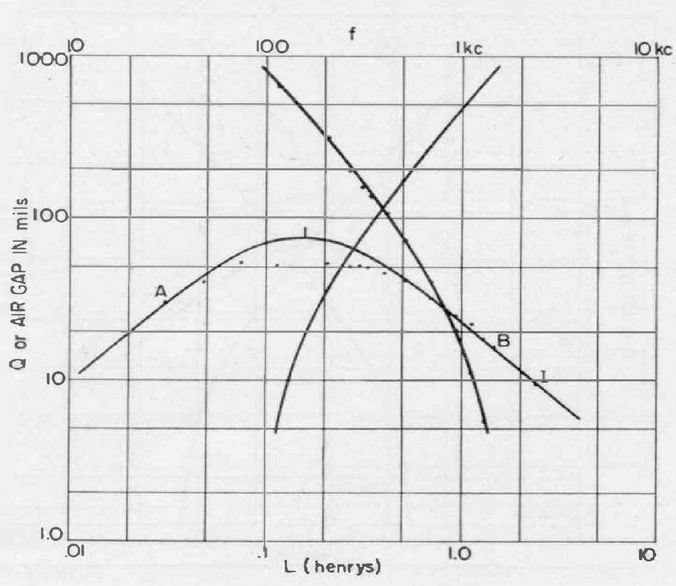


Figure 11. The same coil measured at 1 kilocycle.



The original theory[1] included the assumption that eddy-current losses in the copper were negligible. For moderate air gaps, however, this assumption no longer is valid. Unless the air gap is quite small, nearly all the magnetomotive force is concentrated across the gap. This results in excessive fringing at the gap, and the lines of fringing flux cut the copper, inducing eddy currents. The resultant increased loss in the copper reduces the Q below the ideal value to be expected from the shape of the curves. With larger gaps, approaching the length of the center leg, the leakage flux field has expanded considerably, and much of it goes out around the copper. Meanwhile, the ohmic losses in the coil have gone up considerably, owing to the increased current, and are far larger than the remaining eddy-current losses.

From the winding data given in the captions to Figures 9, 10, and 11, it will be seen that the loss increases with increasing wire size and with increasing frequency, as eddy-current loss would be expected to do.

As was done in the case of Figure 9, two extra points were taken, those at the right of point A, to see whether this analysis was correct and the points would gradually approach the humped curve. They did so very nicely on Figure 10 and pretty well on Figure 11, although the right-hand one of the two was somewhat of a sport.

Eddy Currents in Iron

There is one further phenomenon indicated by the plotted points. In Figure 3 the point marked "I" is way off the humped curve. A ready explanation comes to hand. This figure represents data taken on a core of A-metal laminations. These laminations had no insulating coating to prevent interlamination eddy currents in the way that the scale on silicon steel acts. It is believed, therefore, that there were at the time of measurement excessive interlamination eddy currents. These acted to push the point away from the curve by two effects. First, the extra losses reduced the Q, which pushed the point down. Second, the extra eddy currents reduced the inductance of the coil, thus pushing the point to the left.

Over-all Results

It can now be seen what a considerable amount of information has been made available as a result of a single series of measurements, with varying air gaps in the center leg at a single frequency. The Q_m of the magnetic structure has been determined, from which the Q_m for other alloys and thicknesses of laminations can be easily deduced by methods given in the March, 1942, paper. The second or downward-sloping curve, when taken in conjunction with the known number of turns of the coil used in making the measurements, gives all the information needed to choose the number of turns for a coil of any inductance (at initial permeability) with a given air gap. The third or upward-sloping curve tells what air gap to choose in order to get the maximum Q at any particular frequency. Once a specific design has been determined by use of the second and third curves, its Q behavior at all frequencies can be forecast by using the cardboard template, as suggested in a prior paragraph and more fully covered in the March, 1942, article.

The method of securing information suggested in this article is much simpler than that used in the prior one, since data need be taken for only one instead of several humped Q curves. Lest it be thought that the more complicated method used to secure the information diagrammed in Figure 1 of the March, 1942, article was unwarranted, it should be noted that the simplicity of this present method is dependent on facts adduced therein. If it had not already been demonstrated by actual measurement that the Q_m of a coil occurred at a frequency f_m which varied inversely with the inductance, that is, was higher with larger air gaps, the construction of the third curves from the second ones, purely mathematically, could not have been justified.

[1] Loc. cit.

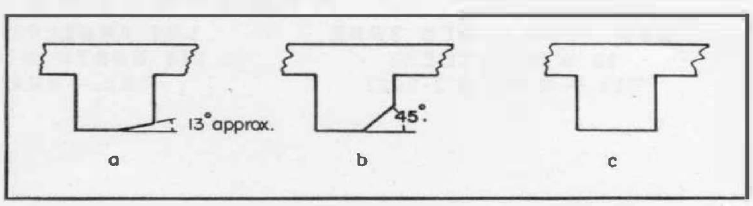


Figure 12. Types of laminations: (a) blunt-angled; (b) sharp-angled; (c) flat.

Example of Use of Complete Theory

An example might be given of a place where the information just described would be needed and used. Suppose it was desired to build a series of high-pass filters having the same impedance and attenuation characteristics but with a series of cut-off frequencies. Figure 13 shows the simple "T" configuration of such a filter. The requirement of similar attenuation characteristics must be interpreted as requiring the same Q for all the coils. By the rules of filter design, the inductance is inverse with the cut-off frequency of the filter. By the rules which have just been developed, the permeability (or inductance) of a coil should be inverse with the frequency at which its Q_m is to occur. Since the Q_m should occur in all filters with the same relation to the cut-off frequency, that is, approximately at cut-off frequency, the conclusion is that the design of this group of filters is quite simple.

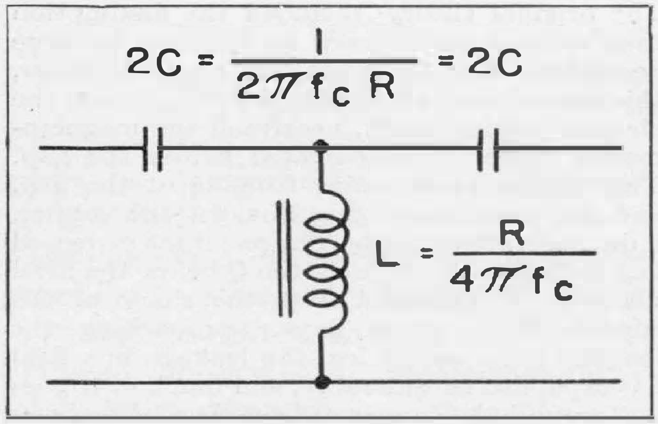


Figure 13.

The same coil winding can be used for all of them (barring unforeseen difficulty). The only differences between coils for filters of different frequencies will be the center-leg air gaps in those coils. The capacitance values will vary inversely with the frequency. The number of different coils required is thus kept at a minimum, although the laminations must be assembled into them differently.

— P. K. McElroy

GLOSSARY

f = frequency of alternating voltage and current.

L = inductance of coil; henrys.

Φ = total r-m-s flux in the iron; maxwells.

μ = incremental permeability (effective) of magnetic circuit.

t = length of average turn; cm.

N = number of turns of wire.

$S = N \cdot \frac{\pi d^2}{4}$ = effective window area (total copper cross section); cm².

δ = lamination thickness; cm.

A = total geometric cross section of magnetic path; cm².

α = stacking factor of iron; dimensionless (ratio of effective area of core material to inside area of coil tube; deficiencies are occasioned by scale, burrs, bent laminations, core-plating, etc.).

l = mean length of flux path; cm.

ρ_c = resistivity of copper; ohm-cm.

ρ_i = resistivity of the lamination material; ohm-cm.

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